Now a function I exists such that $\mathfrak{S}\mathrm{IP}_k^2 = 1$, $\mathfrak{S}\mathrm{IP}_l\mathrm{P}_k = 0$, where $\mathfrak{S}$ denotes a summation throughout the region. Therefore

$$\mathfrak{S}\,\mathrm{I}\,(\phi_{m+1}-\phi_u)^2 = \Sigma[\mathrm{A}_k(1-\alpha_1^{-1}\lambda_k^2)\ldots(1-\alpha_m^{-1}\lambda_k^2)]^2.$$

Now by a sufficient number of suitably chosen α's the polynomial in λ^2 on the right can be made small throughout the range from λ_1^2 to λ_n^2. Therefore the error of ϕ_{m+1} can be made small; for, since I is one-signed, it is measured by the L.H.S. The process is arithmetical.

Under certain conditions the error due to finite central differences is of the form $e_2h^2+e_4h^4+e_6h^6+$etc., where h is the co-ordinate difference and the e's are functions of position independent of h. If the integral has been found for two or more sizes of h, more exact values of it can be extrapolated by this formula.

These methods have been applied in the paper to calculate the stress-function in a masonry dam.

The Initial Accelerated Motion of Electrified Systems of Finite Extent, and the Reaction produced by the Resulting Radiation.

By George W. Walker, A.R.C.Sc., M.A.

(Communicated by Prof. Sir J. Larmor, Sec. R.S. Received December 17, 1909, —Read January 27, 1910.)

(Abstract.)

When the exact equations of motion of a dynamical system are known, it is in general possible, by a well-known process, to determine the initial mode of change from a steady to a variable state, even when the primary equations cannot be completely integrated in the general case. The primary object of this investigation was to show that the same process is applicable to the equations of motion of finite electrified systems. That the results would be applicable to the question of the electric inertia of "electrons" was constantly kept in view; and it was felt to be undesirable to make any approximation depending on extreme smallness of the electrified system.

The various expressions hitherto used for the electric inertia of "electrons," *at speeds comparable with that of radiation*, depend, I think, without exception, on consideration of the energy of a steady state. The "quasi-stationary" principle assumes that, when the energy in a steady state is known, it is

possible to draw true inferences as to the motion of a slightly disturbed system. The logical incorrectness of this has been generally recognised, but there appears to have been a hope that the inferences would not be far from the truth. The principle involves two fallacies: (1) It practically involves the fallacy of differentiating an expression which is true only for a steady state. (ii) It involves the fallacy that when the energy of a steady state is found to contain higher powers of the velocity than the squares, the usual process of getting an equation of motion is valid. It may, of course, happen that a correct result is obtained, but the correctness is only proved by a correct process. The possible failure of the quasi-stationary principle is readily shown by consideration of a dynamical case. When a solid body moves through a gas, any change of its velocity sets up a disturbance radiated into the gas. This disturbance is exactly what is neglected in the quasi-stationary principle, because the energy of this disturbance is not properly included in the energy of a steady state. In the electrical case a similar disturbance occurs, but compared with the dynamical example the motion in the medium is now the dominating influence.

It seems clear that, to get correct results one must proceed from fundamental equations by what I venture to call the Newtonian method of building up equations of motion. Prof. Love's method of analysing the electric vibrations on a sphere supplied the necessary weapon in this method of attack.

The results show that the initial motion from rest in the case of an electrified sphere is governed by two linear equations. Under certain limitations, they lead to the single equation of motion of an "electron" proposed by Lorentz. They thus include the circumstances under which his equation is valid, but possess a greater generality which becomes important in optical applications.

In passing to the case of disturbance from the steady motion at high speed, a difficulty arose as to surface conditions. The point was whether the tangential component of ætherial electric force or of electrodynamic force was continuous at a surface separating a charged body from free æther. It was found to be impossible to arrive at a definite conclusion as to which was correct. There appear to be arguments for and against each view; and so the consequences of each condition have been examined. The new method, using the first of these conditions, confirms Thomson's expression for transverse inertia, while with the second condition we find confirmation of Abraham's expression for longitudinal inertia of a "rigidly" electrified sphere, but not of his expression for transverse inertia. These results for a conducting sphere are compared with the experimental results obtained by

Kaufmann in the case of Becquerel rays. Analysis of the experiments shows excellent agreement with Thomson's formula, but inferior agreement with Abraham's formula or the corrected expression proved by the new method. It appears, however, that a considerable proportion of inertia of ordinary kind is indicated by Kaufmann's experiments.

The method was applied to the linear and rotary accelerated motion of charged insulating spheres at slow speeds. It is shown that if m' is the initial electric inertia for linear motion, the arrangement $\frac{1}{2}m'$ at the centre of the sphere and $\frac{1}{2}m'$ uniformly distributed over the surface gives the inertia effect in rotational motion.

Passing to vibratory motion, we still have two linear equations to determine the motion of a charged conductor, and four linear equations in the case of a charged insulator. Examination shows several new features which appear to have a bearing on the problem of fluorescence. It appears that a charged particle under the influence of incident periodic radiation absorbs the radiation for a certain range of wave-length, and outside this range simply radiates it. General scattering of the incident radiation has no special new significance, but the possibility of absorption raises an interesting point. Accumulation of the energy could not proceed indefinitely, and re-emission must supervene, although the equations do not show how. Doubtless the first approximation ceases to be sufficiently accurate. It is of interest that with present estimates only positively charged ions of molecular dimensions are likely to produce observable effects.

Lastly, the effect of speed of a moving system on the frequency and damping of fundamental vibrations is considered. The results indicate a possible bearing on the problem of luminosity.

Note.—The paper was practically completed towards the end of 1907, but publication has been delayed for private reasons. During the interval I have had the advantage of consulting Sir Joseph Larmor on the points involved. While he is in no way responsible for any of the views expressed in this paper, his valuable suggestions and criticisms have enabled me to remove many obscurities in the work, and I desire most gratefully to record my appreciation of his encouragement and interest.